to predict the surface property distributions given by available exact numerical solutions[9, 10] within 15 to 25%. A more detailed discussion and evaluation of this work will be given in a future paper.

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Effect of Surface Shear on Buckling of Cylindrical Shells

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In this note, a thin-walled circular cylindrical shell is assumed to be under surface shear loading in the longitudinal direction (Fig. 1). When the surface shear τ varies with x only, the additional compression at one end of the cylinder is

$$P_1 = 2\pi R \int_0^l \tau(x)\, dx$$

By the principle of superposition, the shear τ can be considered as a combination of two parts. Referring to Fig. 2,

$$\tau = \tau_1 + \tau_2 \tag{1}$$

In the present case

$$\tau_1 = \tau_2 = \tau/2 \tag{2}$$

The role of τ_1 can be considered that of a body-force component in x direction. Hence, the equilibrium conditions in x and y directions are, respectively,

$$\begin{aligned}(\partial\sigma_x/\partial x) + (\partial\sigma_{xy}/\partial y) + (2\tau_1/t) &= 0\\ (\partial\sigma_y/\partial y) + (\partial\sigma_{xy}/\partial x) &= 0\end{aligned} \tag{3}$$

The potential function V is introduced such that

$$\begin{aligned}\partial V/\partial x &= -(\tau/t)\\ \partial V/\partial y &= 0\end{aligned} \tag{4}$$

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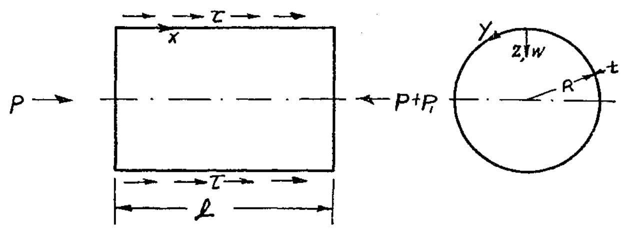

Fig. 1 Cylinder under longitudinal shear and axial compression

When Eqs. (4) are substituted into Eqs. (3) and the terms due to large deflection in the radial direction are included, the compatibility equation has the following form:

$$\nabla^2(\sigma_x + \sigma_y) = (1 + \nu)\nabla^2 V + f(w) \tag{5}$$

In Eq. (5)

$$f(w) = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right) - \frac{1}{R}\frac{\partial^2 w}{\partial x^2}\right] \tag{6}$$

where ν is the Poisson ratio, w the radial deflection, and ∇^2 the Laplacian operator.

The stress function $\varphi(x,y)$ is defined by

$$\begin{aligned}\sigma_x - V &= \partial^2\varphi/\partial y^2\\ \sigma_y - V &= \partial^2\varphi/\partial x^2\\ \sigma_{xy} &= -(\partial^2\varphi/\partial x \partial y)\end{aligned} \tag{7}$$

From Eqs. (7) and (5), the compatibility equation becomes

$$\nabla^4\varphi = -(1 - \nu)\nabla^2 V + E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) - \frac{1}{R}\frac{\partial^2 w}{\partial x^2}\right] \tag{8}$$

The equilibrium condition in the radial direction and the equilibrium relations of moments are found by modifying those given in Ref. 1. These relations are

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + t\left[\sigma_y\left(\frac{1}{R} + \frac{\partial^2 w}{\partial y^2}\right) + \sigma_x\frac{\partial^2 w}{\partial x^2} + 2\sigma_{xy}\frac{\partial^2 w}{\partial x \partial y}\right] - 2\tau_1\frac{\partial w}{\partial x} = 0 \tag{9}$$

$$\frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} - Q_x - \tau_2 t = 0 \tag{10}$$

$$\frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} - Q_y = 0 \tag{11}$$

From Eqs. (9–11), the equilibrium equation can be expressed as

$$D\nabla^4 w = t\left[\sigma_y\left(\frac{1}{R} + \frac{\partial^2 w}{\partial y^2}\right) + \sigma_x\frac{\partial^2 w}{\partial x^2} + 2\sigma_{xy}\frac{\partial^2 w}{\partial x \partial y}\right] + \frac{t^2}{2}\nabla^2 V + t\frac{\partial V}{\partial x}\frac{\partial w}{\partial x} \tag{12}$$

The solution can be found from coupling Eq. (8) with Eq. (12). It should be noted that these equations are analogous

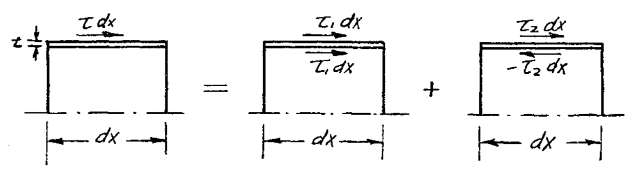

Fig. 2 Equivalence of shear forces (t = thickness, R = radius of cylinder)

to the thermoelastic problems of thin cylindrical shells. When the shear τ is constant, when the effect due to τ_2 [see Eq. (10)] is neglected, and when the radial deflection is small, the problem becomes the one solved in Ref. 2.

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Comparison of Theory with Experiment on a Blunt Axisymmetric Body in Hypersonic Flow

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Nomenclature

a = nondimensional shock layer thickness parameter
a^* = the reference value of a when the parameter $Re_R/M(C)^{1/2}$ is very large
C_H = surface heat transfer coefficient $q/\rho_\infty u_\infty(H_\infty - H_w)$
C_D = nose drag coefficient
C = Chapman-Rubesin constant
D = diameter of model
H = total specific enthalpy
I = $[0.664 + 1.73(T_w/T_0)]$
K = value of p/p_s as $y/y_s \to 0$
M = freestream Mach number
p = pressure
R = radius of model
Re_R = Reynolds number — $\rho_\infty u_\infty R/\mu_\infty$
T = temperature
t = half the thickness of the nose
u = velocity parallel to model axis of symmetry
x,y = rectangular coordinates in direction parallel and normal, respectively, to the model axis of symmetry, with the origin at the nose shoulder
γ = specific heat ratio
ϵ = $\gamma - 1/\gamma + 1$
μ = viscosity coefficient of the gas
ρ = density

Subscripts

b = edge of the effective wall
e = outer edge of the entropy layer
s = immediately behind the shock wave
0 = stagnation conditions on the body
w = wall conditions
∞ = freestream conditions

RECENTLY there have been two new approaches to the solution of the two-dimensional blunt leading edge problem over a flat plate.[1, 2] The use of these developments to predict the heat transfer over a blunt flat plate has the definite advantage of not requiring an experimentally determined pressure distribution. The extension of these theories to the axisymmetric case and comparison with experiment was presented in Ref. 3. This note is a continuation of Ref. 3 specifically dealing with the extension of Ref. 2 to the axisymmetric case.

Cheng et al.[1] obtained a basic differential equation for y_e in terms of the nose drag coefficient for both the two-dimensional and axisymmetric cases valid in the limit of small ϵ. A solution was obtained for the two-dimensional case only. The solution of the axisymmetric problem was obtained in Ref. 3. Once y_e is known, the pressure and heat transfer distributions can be determined by

$$p_w/p_\infty = \gamma M^2[y_e'^2 + (y_e y_e''/2)]$$

$$M^3C_H = 0.332\, M^3(C)^{1/2}/(Re_R)^{1/2}(p_w/p_\infty) \times \left[\int_0^{x/R} p_w/p_\infty d(x/R)\right]^{-1/2}$$

Oguchi's two-dimensional theory[2] can be extended to the axisymmetric case in the following manner. The basic equation in Oguchi's paper (Eq. 3.4) relating the entropy layer thickness to the pressure in the entropy layer can be obtained by writing an approximate mass balance in the entropy layer:

$$\int_0^{y_e} \rho u dy \approx \rho_\infty u_\infty t$$

Assuming that the pressure is constant through the entropy layer and the velocity is approximately equal to the freestream velocity (which was also done in Oguchi's analysis) the forementioned equation can be reduced to

$$y_e = (p_0/p_e)^{1/\gamma}\, \epsilon t$$

which is the same expression obtained by Oguchi. Proceeding in the same manner for an axisymmetric body of constant radius the corresponding relation is

$$y_e^2 = (p_0/p_e)^{1/\gamma}\, \epsilon R^2$$

provided that R^2 can be neglected compared to y_e^2. For an axisymmetric body the shock shape is given by

$$y_s/2R = a(x/2R)^{1/2}$$

Using the hypersonic small disturbance theory result that

$$p_s/p_0 = (1/K)(p_e/p_0)$$

and a pressure law of the form

$$(dy_s/dx)^2 = p_s/p_0$$

the forementioned relations can be combined to give the pressure on the body. It was assumed that the boundary layer displacement correction obtained in Ref. 2 for the two-dimensional case also applies to the axisymmetric case by the use of Mangler's transformation. Adding this in the resulting expression for the pressure is

$$p_w/p_\infty = [\gamma/2(\gamma + 1)]Ka^2M^2(x/2R)^{-1}$$

where

$$a = a^*\left[1 + \delta a^{*[(2-\gamma)/2]} \frac{M(C)^{1/2}}{(Re_R)^{1/2}} I \left(\frac{x}{2R}\right)^{(\gamma-1)/\gamma} \times \left(\ln\frac{x}{2R}\right)^{1/2}\right]^{\gamma/2(\gamma+1)} \left(\frac{x}{2R}\right)^{(1-\gamma)/2(\gamma+1)}$$

and

$$\delta = [\gamma/(\gamma + 1)]^{1/2}\, (\tfrac{1}{4})^{(1-\gamma)/\gamma}\, K^{(2-\gamma)/2\gamma}$$

and

$$a^* = (4/K)^{1/2(\gamma+1)}\ (\epsilon/4)^{\gamma/2(\gamma+1)}\ (y_s/y_e)^{\gamma/(1+\gamma)}$$

over a range where

$$(1 - \gamma)/2(1 + \gamma)\ \ln|x/2R| \ll 1$$

and a/a^* close to one. If the two-dimensional continuity relation were applied to the axisymmetric case (neglecting the effect of transverse curvature and thus assuming y_e is of the order R), the resulting value for a is given in Ref. 3. This approach gives approximately the same value for a^* as the

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